

APPROXIMATE CALCULATION OF PARAMETERS AT  
THE THROAT OF A HYPERSONIC WAKE

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§1. When a blunt body moves at high speed the perturbed region of the flow is a mixture of gases formed by physicochemical transformation. It is very difficult to calculate such a complex region as the near wake under such conditions.

The near wake is a comparatively short region where rapid changes occur in the gas streamline direction under the substantial influence of viscosity. It is therefore postulated that the distribution of pressure, mean-mass velocity, and enthalpy are affected predominantly by dynamic and thermal effects, without allowance for the effects of chemical transformations, and computation of the near wake is performed in two stages. In the first stage we calculate the distributions of pressure, velocity, and enthalpy in the near wake, with the true flow of the gas mixture replaced by an effective perfect gas flow. In the second stage we calculate the composition of the gas mixture and the temperature along the streamlines (or surfaces) for fixed distributions of pressure, velocity, and enthalpy (as determined in the first stage).

In calculating the near wake for a perfect gas we consider three zones (Fig. 1): the zone of upstream influence of base pressure (enclosed between sections 0-2 and 3-4), a constant-pressure mixing zone (between sections 4-5 and 6-7), and a compression zone (between sections 6-7 and 9-10). At section 0-1 we consider all the parameters to be unknown from calculation of the boundary layer on the body.

The compression zone is considered to be a region of interaction between the viscous and the inviscid flow. The viscous flow is described by the laminar boundary-layer system of equations. The dynamic viscosity is calculated from the relation  $\mu/\mu_e = h/h_e$ , where  $h$  is the enthalpy; the subscript  $e$  refers to the outer edge of the wake. A multimoment integral method [1] is used for the calculation. Here the velocity and enthalpy profiles are given in the form of polynomials of second degree in the transformed radial variable

$$\xi = \left( \int_0^r \rho r dr / \int_0^\delta \rho r dr \right)^{1/2},$$

where  $\delta$  is the wake radius. The inviscid flow is calculated approximately from the relations for one-dimensional isentropic motion of a perfect gas. The molecular weight  $m$  of the effective perfect gas flow and its adiabatic index  $\kappa$  are calculated from the relations

$$m = m_2 \left( \sum_{i=1}^N \xi_{i2} / m_{i2} \right)^{-1}; \tag{1.1}$$

$$\kappa = \left( 1 - \frac{RT_2}{m_2 h_2} \right)^{-1}, \tag{1.2}$$

where  $R$  is the universal gas constant;  $T$  is the temperature (the subscript number here and below corresponds to the point number on Fig. 1). The last relation was obtained from the condition that the enthalpy of the effective inviscid flow of a perfect gas should be the same as that of the mixture of gases at point 2 (this is calculated below).

To calculate the mixing zone we use an approximate solution of the system of equations for the constant-pressure laminar mixing layer, as derived in [2]. The solutions for the mixing and compression zones are matched at section 6-7 (whose coordinates are determined during solution of the total problem), using the continuity of the total thickness of the viscous layer at this section and the radial coordinate of the dividing streamline 4-6-8, of all parameters at the outer boundary (at point 7), and of the velocity and enthalpy on the dividing streamline (at point 6).

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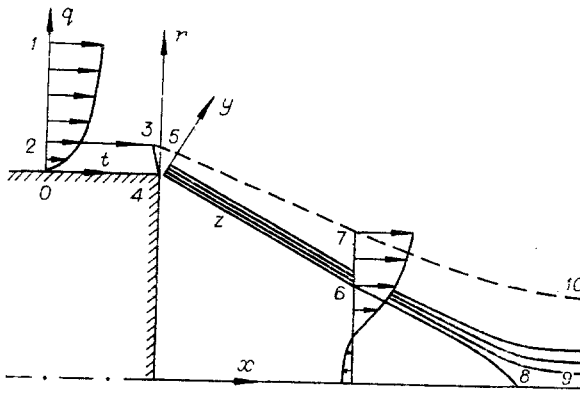


Fig. 1

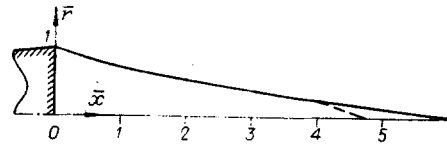


Fig. 2

The initial thickness of the mixing layer is determined by equating the displacement thickness of the unperturbed boundary layer and the mixing layer:

$$\int_{q_0}^{q_1} \left(1 - \frac{\rho}{\rho_1} \frac{v_x}{v_1}\right) dq = \int_{y_1}^{y_2} \left(1 - \frac{\rho}{\rho_2} \frac{v_x}{v_2}\right) dy. \quad (1.3)$$

Then point 2 inside the boundary layer is determined by equating the mass flux passing through sections 0-2 and 4-5:

$$\int_{q_0}^{q_2} \rho v_x dq = \int_{y_4}^{y_5} \rho v_x dy. \quad (1.4)$$

The gas parameters at point 3 are determined as follows. According to the data of [3], the pressure drop in the wall layer near the corner point is such that the wall streamline reaches the corner point with sonic speed. Using this result and allowing for the fact (which follows from the system of equations for the inviscid region describing flow near a corner point [3]) that in this region

$$\partial p / \partial q = 0,$$

we determine that the pressure at section 3-4 is constant and that the pressure drop between sections 0-2 and 3-4 is equal to the critical value,

$$\frac{p_1}{p_3} = \left(\frac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}}.$$

The velocity and enthalpy at point 3 are calculated from the relations for isentropic motion of a gas, allowing for this pressure drop.

The unique solution of the near-wake problem is derived from the condition that the solution must pass through the singular saddle point of the system of differential equations for the compression zone.

§2. The fields for concentrations of individual components and the temperature of the mixture of gases in the viscous near wake are calculated along streamlines. Here the fields of pressure  $p$ , vectorial components  $v_x$ ,  $v_r$ , and enthalpy  $h$  are regarded as given. These quantities are taken from calculation of the near wake according to the scheme described in Sec. 1. The profile of relative mass concentration of the  $i$ -th component at the initial section (4-5) of the constant-pressure region is given in the form of a second-degree polynomial:

$$\begin{aligned} \xi_i(y) &= a_i + b_i \sigma + c_i \sigma^2, \\ \sigma &= \int_0^y \frac{\rho}{\rho_5} dy \bigg/ \int_0^{y_5} \frac{\rho}{\rho_5} dy. \end{aligned} \quad (2.1)$$

The length of the upstream base pressure influence zone is small, amounting only to several boundary-layer thicknesses [3]. In the region between sections 0-2 and 4-5 there is a sharp pressure drop from the value on the body ahead of the base. Therefore, we can consider that the chemical reactions are frozen in the motion of the mixture in this region. In this case the number of particles between these sections in the flow must be equal:

$$\int_{q_0}^{q_1} n_i v_i dq = \int_{y_4}^{y_5} n_i v_z dy. \quad (2.2)$$

With the requirement that  $\xi_{i4} = \xi_{i0}$ ,  $\xi_{i5} = \xi_{i2}$ , and allowing for Eq. (2.2), we have three conditions for calculating the coefficients of the polynomials (2.1)  $a_i$ ,  $b_i$ ,  $c_i$ . Thus, the concentrations of individual components at the initial section of the mixing layer are determined. The temperature at this section is calculated from the relation

$$\sum_{i=1}^N h_i(T) \xi_i = h, \quad (2.3)$$

where  $h_i(T)$  is the enthalpy of the  $i$ -th component.

The first stream tube is extended along the dividing streamline 4-6-8 and later along the wake axis. Its thickness is determined by the condition that the mass in the stream tube be constant. A second stream tube is drawn out along the boundary of the first, and so on.

Along the stream tubes the equations for conservation of concentrations of the individual components

$$\rho_j v_j d\xi_{ij}/dS = W_{ij}$$

are integrated, where  $j$  is the stream tube number;  $S$  are coordinates reckoned along the tubes;  $W_{ij}$  is the rate of conversion of the  $i$ -th component as a result of physicochemical transformations. The value of  $W_i$  is calculated from the relation

$$W_i = m_i \sum_{k=1}^s (v_{ik}^* - v_{ik}) \left[ K_{fk}(T) \prod_{j=1}^N n_j^{v_{jk}} - K_{bk}(T) \prod_{j=1}^N n_j^{v_{jk}^*} \right],$$

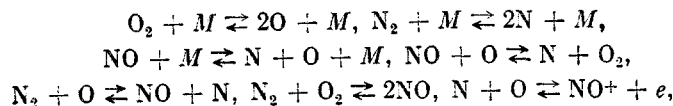
where  $s$  is the number of reactions;  $v_{ik}$  and  $v_{ik}^*$  are the stoichiometric coefficients for the  $i$ -th component in the  $k$ -th reaction in the forward and reverse directions, respectively; and  $K_{fk}$ ,  $K_{bk}$  are the rate constants for the forward and reverse reactions.

The temperature is calculated from Eq. (2.3) and the mass density is calculated from the equation of state

$$\rho_j = p_j / \left( RT_j \sum_{i=1}^N \xi_{ij}/m_i \right).$$

The calculation is carried out up to section 9-10.

§3. We now present the computational data referring to the two versions of the motion of a blunt cone in air. The calculation is based on the following system of physicochemical processes:



where  $M$  is the third particle. The rate constants for these reactions were taken from [4].

The boundary-layer parameters at section 0-2 were calculated approximately using the following technique.

1. The stream tube method of [5] was used to calculate the inviscid nonequilibrium flow over the body.
2. The data of [6] were used to calculate the relative mass flow rate of gas

$$Q = \frac{\int_{q_0}^{q_1} \rho v_i r_i dq}{\pi r_c^2 \rho_\infty v_\infty},$$

where  $r_c$  is the nose radius and the subscript  $\infty$  refers to the parameters of the undisturbed stream.

3. It is assumed that the velocity profile in the boundary layer on the conical body surface is given by the Blasius function

$$v_i/v_1 = \varphi'(\eta), \quad (3.1)$$

where  $\varphi(\eta)$  is the solution of the Blasius problem

$$2\varphi'''' + \varphi\varphi'' = 0, \quad \varphi(0) = \varphi'(0) = 0, \quad \varphi'(\infty) = 1,$$

the relation between the total enthalpy and the velocity is given by the Crocco integral

$$H = (H_1 - H_0) \frac{v_t}{v_1} + H_0, \quad H = h + v_t^2/2, \quad (3.2)$$

and the relation between the physical coordinate  $q$  and the Blasius coordinate  $\eta$  under these assumptions will take the form

$$\frac{q}{r_4} = \frac{Q\rho_\infty v_\infty}{2\varphi(\eta_1)\rho_1 v_1} \left(\frac{r_c}{r_4}\right)^2 \left\{ (1+d) \left[ \frac{H_0}{H_1} \eta + \left(1 - \frac{H_0}{H_1}\right) \varphi(\eta) \right] - d [\varphi(\eta) \varphi'(\eta) + 2\varphi''(\eta) - 2\varphi''(0)] \right\}, \quad (3.3)$$

where

$$d = \left\{ \left( \frac{v_\infty}{v_1} \right)^2 \left[ \frac{2}{(\alpha_\infty - 1) M_\infty^2} + 1 \right] - 1 \right\}^{-1}.$$

The boundary-layer thickness is calculated from Eq. (3.3) with  $\eta = \eta_1 = 5$  ( $\varphi(\eta_1) = 3.28$ ,  $\varphi'(\eta_1) \approx 1$ ,  $\varphi''(\eta_1) \approx 0$ ).

4. To evaluate the concentrations of individual components at point 2 we assume that the concentration profiles are linear near the walls. The transverse derivative of concentration at the wall is evaluated from the simple solution given in [7] and valid for the case of binary diffusion and chemical reactions:

$$\begin{aligned} \frac{d\xi_i}{d\eta} &= \frac{Sm_i^{1/3}}{2.41 \sqrt{2}} (\xi_{i1} - \xi_{i0}), \quad i = 1, 2, \dots, N, \\ \xi_i &= \frac{Sm_i^{1/3}}{2.41 \sqrt{2}} (\xi_{i1} - \xi_{i0}) \eta + \xi_{i0}, \quad \eta \leq 1.5, \end{aligned} \quad (3.4)$$

where  $Sm_i$  is the Schmidt number for the  $i$ -th component. All the parameters with subscripts 1 are determined from calculation of the inviscid nonequilibrium flow over the body, allowing for nonuniformity of the inviscid flow. Thus, in this approach to calculating the concentrations inside the boundary layer, the nonequilibrium nature of the physicochemical transformation is taken into account only via its influence on the inviscid region parameters.

The Schmidt number in the calculation was assumed to be 0.7 for the neutral components and 0.35 for the charged components.

Relations (1.3), (1.4), and (2.3)–(3.4) were used to calculate the coordinates of point 2 and the values at that point of the velocity, enthalpy, temperature, and concentrations of the individual components. Equations (1.1) and (1.2) were used to calculate the adiabatic index of the effective perfect gas flow.

For simplicity in the calculations, the wake was assumed to be isoenergetic with a total enthalpy of  $H_2$ .

Figures 2–4 show the results of calculating the parameters behind spherically blunted cones with semi-vertex angle  $10^\circ$ , moving in the earth's atmosphere at altitude 50 km with surface temperature  $1000^\circ\text{K}$ . The solid curves in Figs. 2–4 refer to the case  $r_c = 0.7$  m,  $r_4 = 1$  m,  $v_\infty = 5.5$  km/sec, and the dashed curves refer to the case  $r_c = 0.15$  m,  $r_4 = 0.5$  m,  $v_\infty = 6.5$  km/sec. Figure 2 shows the dividing streamline. The notation is  $\bar{x} = x/r_4$ ,  $\bar{r} = r/r_4$ . Figure 3 shows the distributions of temperature (curves 1) and electron density (curves 2) along the dividing streamline. Figure 4 shows the profiles of these parameters in the section containing the rear stagnation point.

The values of enthalpy obtained in the calculation at the rear stagnation point, referred to the total enthalpy of the incident flow, are 0.274 and 0.328. The corresponding values of equilibrium temperature are 2500 and  $2950^\circ\text{K}$ , which differ appreciably from the data of Figs. 3 and 4. Hence, it follows that the influence of the nonequilibrium nature of the physicochemical processes in the viscous near wake is appreciable.

Consequently, the approximate method considered is based on computing only the decisive effects, i.e., the effects of viscosity and heat conduction (Sec. 1) and of the nonequilibrium nature of the physicochemical transformations (Sec. 2). Since there are at present no experimental data or data of more accurate calculations of the composition of gas mixtures in the near wake to justify this technique, we are restricted only to the statement that the solution of the dynamic problem agrees with the data of other papers.

The main difference of the methods considered in Sec. 1 from methods suggested in [1, 8] is, first of all, in calculating the influence of base pressure upstream and, secondly, in using conditions (1.3) and (1.4) to

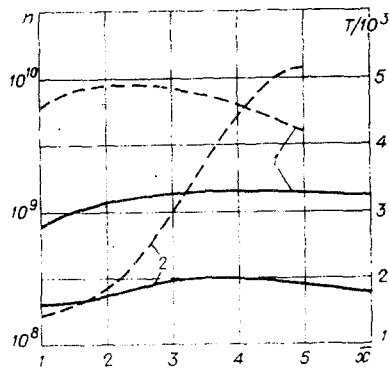


Fig. 3

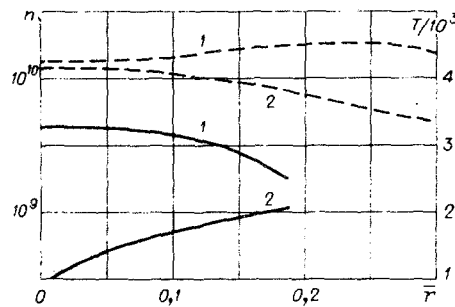


Fig. 4

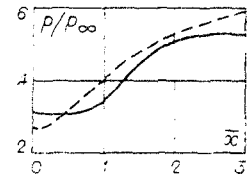


Fig. 5

determine the initial wake thickness. Calculations show that for blunt bodies and large Reynolds number ( $Re_\infty$ ) the base pressure values are close to the corresponding values calculated using the method of [8], which, as was shown in [8], agree with experimental data. However, the present method has a number of substantial advantages. Since we cannot discuss these in detail here, we note only that the calculated values of the mass of gas absorbed by the viscous wake are appreciably lower, which agrees qualitatively with the data of more accurate calculations in [9], and the present method, in distinction from the methods of [1, 8], allows one to calculate the near wake over a considerably wider range of external conditions, and, in particular, in difficult cases like small Reynolds number ( $\sim 10^4$ ) and slender bodies moving at hypersonic speed.

Figure 5 compares the calculated pressure distribution along the wake, obtained using a more rigorous method – numerical integration of the complete system of Navier–Stokes equations for a perfect gas [10] (solid curve). The calculated data refer to a short cone with spherical blunting (semivertex angle of  $10^\circ$ ), moving with  $M_\infty = 15$ ,  $Re_\infty = 1.3 \cdot 10^4$ . In the calculation by the above method the same conditions were given at the initial section 0–1 as in [10] (these data were kindly supplied by N. S. Kokoshinskii). It is clear that there is good interagreement between the computational data.

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